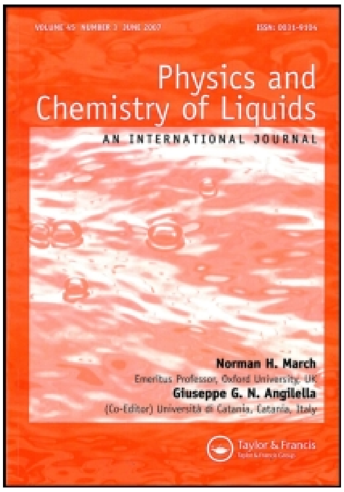

Dependence of Transition Temperature *T_c* of Cuprates on Interlayer Interaction Via Space Charged Boson Binding Energy

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LETTER

DEPENDENCE OF TRANSITION TEMPERATURE T_C OF CUPRATES ON INTERLAYER INTERACTION VIA $\vec{r}$ SPACE CHARGED BOSON BINDING ENERGY

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Starting from Fermi liquid theory at elevated temperatures, March, Pucci and Egorov have recently presented evidence for the existence of charged bosons above T_c in underdoped $YBa_2Cu_4O_8$. Here a theory of T_c is constructed which uses (a) the Bose–Einstein condensation formula, with effective mass m^* and Boson density $n_b(T_c)$, (b) experimental data on plasmons through the transition temperature to extract the temperature dependence of the Boson density $n_b(T)$, and (c) tunneling of holons, with the Bosons assumed to be $\vec{r}$ space biholons, the binding energy of the latter depending on tunneling matrix elements in both intra- and interlayer directions.

KEY WORDS: Fermi liquid theory, antiferromagnetic wave number, biholons.

Recent work by Egorov and March[1] has utilized the results of the 2D Fermi liquid theory of Kohno and Yamada[2], which treats charge carriers flowing through antiferromagnetic assemblies. These latter workers show that the electrical resistivity R can be written in terms of the susceptibility $\chi(\vec{Q})$ at the antiferromagnetic wave vector $\vec{Q}$ as[2]

$$R \propto T^2\chi(\vec{Q}) \tag{1}$$

Secondly, the nuclear spin-lattice relaxation time T_1, the time constant for nuclear spins to readjust to thermal equilibrium with the lattice (i.e., d-spins in the cuprates), can also be expressed as

$$(TT_1)^{-1} \propto \chi(\vec{Q}) \tag{2}$$

As pointed out by Egorov and March[1], division of Eq. (1) by Eq. (2) leads immediately to the prediction

$$RT_1 \propto T \tag{3}$$

This has been tested[1] using experimental data of Bucher *et al.*,[3] on underdoped $YBa_2Cu_4O_8$. While the prediction (3) is well obeyed over a wide temperature range, in this material with $T_c \sim 80$ K there is a clear crossover to a new regime at $T \lesssim 100$ K, after which RT_1 increased with further decrease of temperature. The outcome is that March, Pucci and Egorov[4] have interpreted the new regime below 100 K as due to charged ($2e$) Bosons forming in $\vec{r}$ space, with a binding energy $E_b \simeq 0.01$ eV. These, as pointed out by Nozières and Schmitt-Rink[5], are the precursors of the transition to the superconducting state in the strong–coupling regime.

The purpose of the present work is to apply the above proposals to investigate how the transition temperature T_c can depend on the interlayer coupling in the cuprates, a dependence which is now well established experimentally.

The starting point of the present study is to determine T_c from the Bose–Einstein condensation of the above $\vec{r}$ space Bosons, having a temperature dependent density $n_b(T)$. Then one has for T_c:

$$T_c = \frac{h^2}{2\pi m^* k_B}\left(\frac{n_b(T_c)}{2.612}\right)^{2/3} \tag{4}$$

where evidently the Boson density is required at $T = T_c$. The effective mass m^* is determined by largely intralayer dynamics.

There appear presently to be two feasible approaches to determine the Boson density $n_b(T)$. One is via experiments on plasmons excited in the transverse direction[6], which lead to the measurement of a plasma frequency in both superconducting and normal states; namely $\omega_p(T)$. Using the usual Langmuir formula for the plasma frequency in terms of the charged ($2e$) Bosons, namely

$$\omega_p(T) = \left(\frac{4\pi n_b(T)(2e)^2}{m_\perp}\right)^{1/2}, \tag{5}$$

it is clear that $n_b(T)$ is determined by the measurements of $\omega_p(T)$. For the material studied by Tamasaku *et al.*[6], $n_b(T)/n_b(8\,\mathrm{K})$ is plotted in Figure 1 as a function of temperature through the superconducting state into the normal state.

The second approach is to calculate $n_b(T)$ from the chemical equilibrium requirements that, for doping concentration x, $n_b(T)$ are ($2e$) Bosons and the remaining particles are quasiparticle monomer Fermions of charge e. This can be done by constructing the free energy from a sum of contributions of Bosons, density $n_b(T)$, and Fermions also, of course, from particle conservation, with density also temperature dependent, and decreasing with decreasing temperature, as charged Bosons are formed from dimers of Fermions. This second approach will be discussed elsewhere. However it leads to an approximate form

$$n_b(T) \propto \frac{A}{1+K} \tag{6}$$

where K involves crucially a Boltzmann energy distribution with argument E_b/k_BT, the quantity E_b being the Boson binding energy. The constant A corresponds to $n_b(T)$ in the limit of low temperature. Though, we stress, Eq. (6) is approximate, it suggests plotting $\ln(1 - n_b(T)/A)$ against $1/T$ and this plot has been constructed from Fig. 1. We extract the binding energy E_b from the slope of Figure 2 as $E_b \simeq 7.6 \times 10^{-3}$ eV.

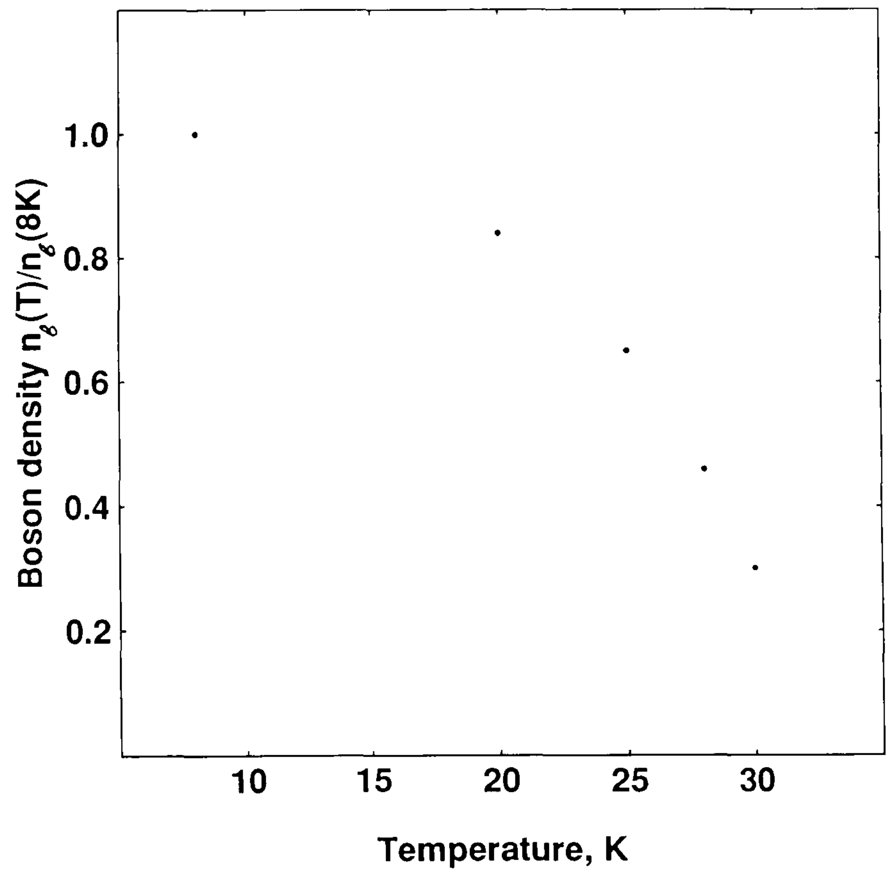

Figure 1 Shows boson density $n_b(T)$ versus temperature T; $n_b(T)$ is normalized to the lowest-temperature (8 K) value for which experimental data[6] on plasma frequency $\omega_p(T)$ is available.

Finally, we come to a central question of the theory: what determines the binding energy E_b? The answer appears to lie in the holon picture. Holons can tunnel between layers, and if the hopping matrix element for this is denoted by $t_\perp$, then for biholons as the $\vec{r}$ space Bosons one can write[7,8]

$$E_b = \frac{Bt_\perp^2}{t} \tag{7}$$

where $B \lesssim 1$. In Eq. (7) $t_\parallel$ is the intraplane hopping matrix element. There seems no difficulty in getting the right order of magnitude for E_b from Eq. (7), with $B \sim 1$.

In summary, T_c is related to the density of $\vec{r}$ space Bosons at T_c. In turn, via Eqs. (5) and (6), $n_b(T)$ is seen to be determined through a Boltzmann factor through a ratio E_b/k_BT_c. For underdoped $YBa_2Cu_4O_8$, this ratio is $\sim 5/4$ from the work of March, Pucci and Egorov. The final step in relating T_c to interlayer coupling is via Eq. (7), where $t_\perp^2$ is evidently sensitive to interlayer coupling.

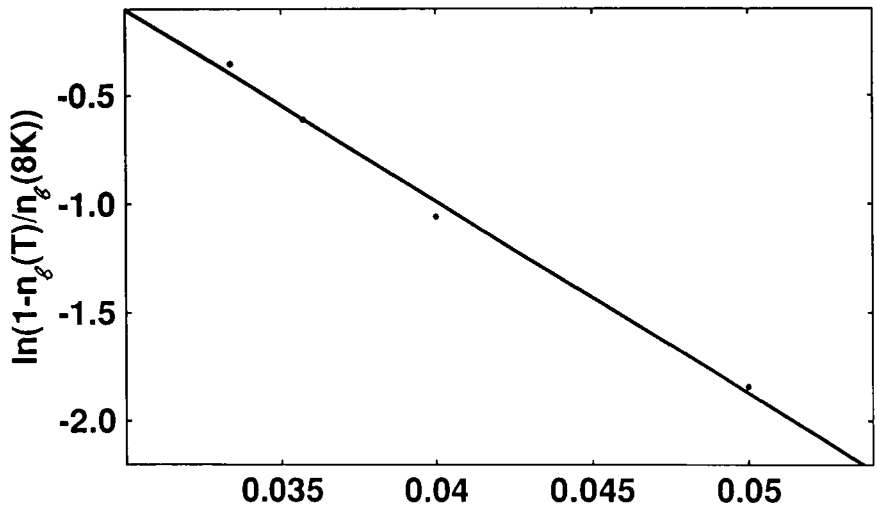

Figure 2 Shows $\ln(1 - n_b(T)/n_b(8\,\mathrm{K}))$ versus inverse temperature $1/T$. The dots correspond to the experimental data of Tamasaku *et al.*[6]; the straight line is the best linear fit to the experimental points.

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